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Use of Catastrophe Theory to Obtain a Fundamental Understanding of Elementary Particle Stability

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Using Arnold's Classification Theorem applied to a four-dimensional manifold, it is shown that there is only a finite number of ways in which energy can discontinuously change state. It is demonstrated that each of these energy flow pathways can be associated with a distinct elementary particle. The theory not only shows how the formation of particles from the stress-energy present in the space-time manifold can be predicted from first principles, but also that there must exist five fundamental forces in a universe in which discontinuous energy transitions are possible. Finally, the existence of a new, as yet undiscovered particle is predicted, which is associated with this new fifth force.

1. INTRODUCTION

For a wide range of processes and interactions, it can be shown that there are a finite number of distinct and stable pathways by which a system can discontinuously change from one state to another in accordance with well-known conservation laws (Rand, 1977). Such transitions have long been known as unfoldings or catastrophes and are characterized by a system suddenly shifting from one stable state to another while the parameters controlling the interaction change continuously. Each of the unfoldings or catastrophes represents a pattern of response set only by the number of control parameters and by the fact that these parameters can be represented by a potential, not by the internal processes that link them to the system.

Thom (1975) defined a catastrophe as any discontinuous transition which takes place when a system can have more than one stable state or stable pathway of change at a given space-time coordinate. The transition is discontinuous not because there are no intervening states or pathways, but because none of them is stable, and, therefore, undetectable. The amount of time spent in the intervening states is bound to be brief compared to

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that spent in the stable states, so no information can be obtained about the status of the system during the catastrophe. The elementary catastrophes are the only permitted ways in which transitions may occur between stable energy states.

It was shown by Thom that in any system governed by a potential and in which the interaction of the system is controlled by no more than five control factors, a finite number of qualitatively different types of discontinuous transitions can take place. If more than five control factors are present, then there is an infinite number of singularities without unique unfoldings, so it becomes impossible to differentiate between the different catastrophes.

In Thom's catastrophe theory the qualitative type of a specific stable discontinuity is not dependent on the particular nature of the potential, nor on the particular conditions controlling the interactions. It is only significant that the potential exists and that the number of control factors can be clearly established. The theory provides a basis for classification, but yields no quantitative information. Such information is already provided by existing theories, such as Einstein's geometrodynamics or Dirac's quantum electrodynamics. Catastrophe theory must therefore be seen as an extension to existing theories, linking them into the realm of discontinuous change (Poston and Stewart, 1978; Woodcock and Davis, 1978).

2. CATASTROPHE THEORY ACTING ON THE SPACE-TIME MANIFOLD

Einstein assumed that physical space could be represented as a curved, four-dimensional space immersed in a flat space of a higher number of dimensions. Using Riemannian geometry, he showed that this physical space can be represented by a network of curvilinear coordinates such that the $g_{\mu\nu}$ of the metric, given as functions of the coordinates, fix all the elements of distance so they determine the coordinate system as well as the curvature of the space.

Einstein's law of gravitation can be written in empty space as

$$R_{\mu\nu} = 0$$

where "empty" indicates that no matter or physical fields are present except for the gravitational field. Quantitative laws, such as those for gravitation and electromagnetism, are inextricably bound up with the geometry of space-time. Thus it can be concluded that if it is desired to develop a quantitative law which unifies all natural phenomena, it is necessary to show that all phenomena are reducible to manipulation of the geometry of space-time.

Taking Einstein's geometrodynamics as the framework within which catastrophe theory must function, one relates the Einstein tensor to the stress-energy manifold by the equation

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

where the stress-energy tensor $T_{\mu\nu}$ is the source term for the curvature of the space-time manifold, and indicates the presence of mass-energy. Physical matter is seen as a collection of elementary particles of identifiable properties. An elementary particle is seen as a structural singularity in the fabric of the space-time manifold, with specific properties and without any internal structure, which can change into another elementary particle of specific properties by undergoing a catastrophe. It is desired to show, from first principles, the stable pathways by which the stress-energy of the space-time manifold can be transformed into elementary particles of known properties. Using catastrophy theory, it is possible to show how discrete quantum-objects, to which physics has assigned the name "particles," can arise as a result of discontinuous energy transitions within the four-dimensional manifold described by Einstein. Rand (1977) showed that all germs of codimension ≤ 5 have been classified. He also showed that all germs of cod $\leq$ 5 are simple and can be represented as

$$A_1^{\pm}, A_2^{\pm}, A_3^{\pm}, A_4^{\pm}, A_5^{\pm}, A_6^{\pm}, D_4^{\pm}, D_5^{\pm}, D_6^{\pm}, E_6^{\pm}, E_7, E_8.$$

Each of these represents a different structural singularity in the fabric of space-time. The fundamental difference between them is the number of control factors or "forces" which govern each discontinuity's interactions with the outside world.

Consider now the experimentally known particles in order of complexity, meaning in increasing order of the number of ways in which they can interact with their environment. The first order particles are those that interact via one pathway. Such a particle, about which more will be said later, is represented by germ E_8. The second-order particles are those that interact via two pathways. These include the photon, which interacts by the electromagnetic and the gravitational forces, and the neutrino, which is governed by the gravitational and the weak interactions. The photon is therefore represented by the germ $E_7 = \gamma^0$. Note that the Classification Theorem indicates that the particle represented by the germ E_6 will have a conjugate, or antiparticle, associated with it. Thus, the neutrino and its antiparticle, the antineutrino, are represented by the germ $E_6^{\pm}$. Note further that since the photon and neutrino are in the same family, according to the Classification Theorem, then this indicates that the weak and electromagnetic interactions must be nothing more than different manifestations of the same fundamental force. This is supported by the experimentally proven electroweak theory. Thus, $E_6^+ = \nu$ and $E_6^- = \bar{\nu}$.

The next higher order of particles are those whose interactions are governed by three control factors, the electromagnetic, weak, and gravitational interactions. This is the lepton family of particles, which include the electron, muon, and τ particles. Thus, the "D" family of germs, noting that provision is made for a full set of antiparticles, appears as follows:

$$D_4^+ = e^-, \qquad D_5^+ = \mu^-, \qquad D_6^+ = \tau^-$$
$$D_4^- = e^+, \qquad D_5^- = \mu^+, \qquad D_6^- = \tau^+$$

It was shown by Thom that in each family of germs, each of the higher order catastrophes is built up out of those of lower order and that they tend to "unfold" discontinuously into a structure of lower order. The inherent instability of the higher order catastrophe structures is verified experimentally by the fact that higher order particles in a particular family decay into lower order members of the same family.

The highest order of particles will undergo the full spectrum of interactions, including the gravitational, electroweak, strong, and, for want of a better definition, hypercharge interactions. This last terminology has been chosen for the fifth type of interaction in order to conform with recent experimental observations suggesting the existence of an as yet undiscovered force which apparently arises from the presence of hypercharge. Thus, the "A" family of germs represents the collection of quarks and their respective antiquarks. They are

$$A_1^+ = u, \qquad A_2^+ = d, \qquad A_3^+ = s, \qquad A_4^+ = c, \qquad A_5^+ = t, \qquad A_6^+ = b$$
$$A_1^- = \bar{u}, \qquad A_2^- = \bar{d}, \qquad A_3^- = \bar{s}, \qquad A_4^- = \bar{c}, \qquad A_5^- = \bar{t}, \qquad A_6^- = \bar{b}$$

where the quarks, represented by their usual symbols, are up, down, strange, charmed, truth, and beauty (or top and bottom). Catastrophe theory again predicts that the higher order quarks will decay into those of lower order by emission of lower order particles.

The theory predicts that all interactions involving discontinuous changes in the stress-energy of the space-time manifold will be governed by no more than five control factors if stable states are to result. For catastrophes with more than five control factors, there is an infinite number of singularities without unique unfoldings, and it is no longer possible to differentiate the possible catastrophe surfaces from each other.

It should be noted here that the neutrinos are not differentiated into the three different ones that are experimentally observed, namely the electron, muon, and τ neutrinos, because these are not considered to be, from the point of view of catastrophe theory, different particles. A similar analogy could be obtained by attempting to differentiate between the structure of a γ-ray and an X-ray photon.

Only one singularity remains unassigned in order to complete this work. The theory predicts that a particle should exist that interacts via only one mode. This particle, which remains to be observed experimentally, would be the carrier of the hypercharge interaction. This chargeless particle, which would have properties similar to those previously predicted for the graviton, would be difficult to observe, since it would only interact by the hypercharge interaction. For purposes of reference in further papers, this particle will be referred to as the $\mathscr{H}$ particle. Thus, $E_8 = \mathscr{H}^0$.

3. CONCLUSION

This work has shown why it has not been possible to "quantize" gravity. The space-time manifold provides the "fabric" from which the quantum objects observed in elementary particle physics are made. Particles represent the discontinuous localizations of stress energy, and their creation and destruction follow specific rules, which can be derived from first principles. It is no longer necessary to try and develop "quantum gravity," because all interactions can be shown to be reducable to space-time geometry.

This paper has shown that there is an infinite number of ways for the stress-energy to change state continuously, but only a finite number of discrete ways in which it can change form discontinuously. It has been shown that each of these discrete energy transformation pathways, while under the influence of between one and five control factors known as "forces," can be seen to result in a specific observable particle with measurable properties and interaction pathways. Thus, it has been shown that the known spectrum of elementary particles can be determined from first principles using the results of catastrophe theory.

Since the classification of the elementary catastrophes depends on the local analysis of the topological properties in the immediate neighborhood of the singularity, nothing can be said about the global behavior of the system. The development of the elementary catastrophes into a global scheme with possible applications to geometrodynamics or black hole dynamics awaits further advances in mathematics.

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